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ESTIMATION OF SPATIALLY AND TIME DEPENDENT SOURCE TERM IN A TWO-REGION PROBLEM

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Abstract – This work deals with the use of the conjugate gradient method with adjoint problem for the estimation of the source term in a two-region problem. The physical problem consists of heat conduction in two contacting rectangular regions, where the source term is known to exist in only one of them. The source term is supposed to vary in time as well as within the region. Simulated temperature measurements are used in the inverse analysis for the estimation of functions containing discontinuities and sharp-corners.

1. INTRODUCTION

Inverse problems dealing with the identification of source terms have been drawing the attention of different research groups for more than a decade, because of its importance in many practical applications, such as predicting and controlling chemical reactions and phase-change, detecting inclusions or non-homogeneities, etc. For the solution of such kind of inverse problem, a variety of methods have been examined [1-17].

This paper is concerned with the estimation of the source term in a two-region heat conduction problem. The physical problem of interest is typical of the manufacturing of Micro-electromechanical Systems (MEMS), or even other macro-scale parts, and involves the pressing of a substrate on a mold for replication purposes that shall undergo a prescribed thermal history [18]. The estimation of the source term within the press by using the temperature variation at selected points within the substrate is under picture in this paper. For the estimation of the source term, no information is *a priori* assumed available regarding its functional form, except for the functional space that it belongs to. It is assumed that the unknown belongs to the Hilbert space of square integrable functions in the spatial and time domains of interest. The unknown source term is estimated by using the conjugate gradient method with adjoint problem formulation [19-21], as described below.

2. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The physical problem considered in this work consists of the two-dimensional heat conduction in two contacting rectangular regions, as depicted in Figure 1. The widths of both regions are a, while the thickness of region A is b and the thickness of region B is $(c\text{-}b)$. Regions A and B are supposed to be initially at the uniform temperatures T_{0A} and T_{0B}, respectively. For $t > 0$, the two regions are put in contact and heat is generated in region B at a volumetric rate $g(x,y,t)$. The contact conductance at the interface between the two regions, h_c, is supposed to be uniform and the other boundaries of regions A and B are supposed to be insulated. By assuming that the thermal properties of regions A and B are constant, the mathematical formulation for this problem is given in dimensionless form as:

<u>Region A:</u>

$$\frac{\partial \theta_A}{\partial \tau} = \frac{\partial^2 \theta_A}{\partial X^2} + \frac{\partial^2 \theta_A}{\partial Y^2} \qquad \text{in } 0 < X < 1 \text{ , } 0 < Y < B \text{ , for } \tau > 0 \tag{1.a}$$

$$\left.\frac{\partial \theta_A}{\partial X}\right|_{X=0} = 0 \qquad \text{at } X = 0 \text{ , } 0 < Y < B \text{ , for } \tau > 0 \tag{1.b}$$

$$\left.\frac{\partial\theta_A}{\partial X}\right|_{X=1}=0 \qquad \text{at } X=1,\ 0<Y<B,\ \text{for } \tau>0 \qquad (1.c)$$

$$\left.\frac{\partial\theta_A}{\partial Y}\right|_{Y=0}=0 \qquad \text{at } Y=0,\ 0<X<1,\ \text{for } \tau>0 \qquad (1.d)$$

$$\left.\frac{\partial\theta_A}{\partial Y}\right|_{Y=B}=Bi(\theta_B-\theta_A) \qquad \text{at } Y=B,\ 0<X<1,\ \text{for } \tau>0 \qquad (1.e)$$

$$\theta_A(X,Y,0)=0 \qquad \text{for } \tau=0,\ \text{in } 0<X<1,\ 0<Y<B \qquad (1.f)$$

<u>Region B</u>

$$\frac{1}{\Lambda}\frac{\partial\theta_B}{\partial\tau}=\frac{\partial^2\theta_B}{\partial X^2}+\frac{\partial^2\theta_B}{\partial Y^2}+\frac{G(X,Y,\tau)}{K} \qquad \text{in } 0<X<1,\ B<Y<C,\ \text{for } \tau>0 \qquad (2.a)$$

$$\left.\frac{\partial\theta_B}{\partial X}\right|_{X=0}=0 \qquad \text{at } X=0,\ B<Y<C,\ \text{for } \tau>0 \qquad (2.b)$$

$$\left.\frac{\partial\theta_B}{\partial X}\right|_{X=1}=0 \qquad \text{at } X=1,\ B<Y<C,\ \text{for } \tau>0 \qquad (2.c)$$

$$\left.\frac{\partial\theta_B}{\partial Y}\right|_{Y=B}=\frac{1}{K}Bi(\theta_B-\theta_A) \qquad \text{at } Y=B,\ 0<X<1,\ \text{for } \tau>0 \qquad (2.d)$$

$$\left.\frac{\partial\theta_B}{\partial Y}\right|_{Y=C}=0 \qquad \text{at } Y=C,\ 0<X<1,\ \text{for } \tau>0 \qquad (2.e)$$

$$\theta_B(X,Y,0)=\theta_{0B} \qquad \text{for } \tau=0,\ \text{in } 0<X<1,\ B<Y<C \qquad (2.f)$$

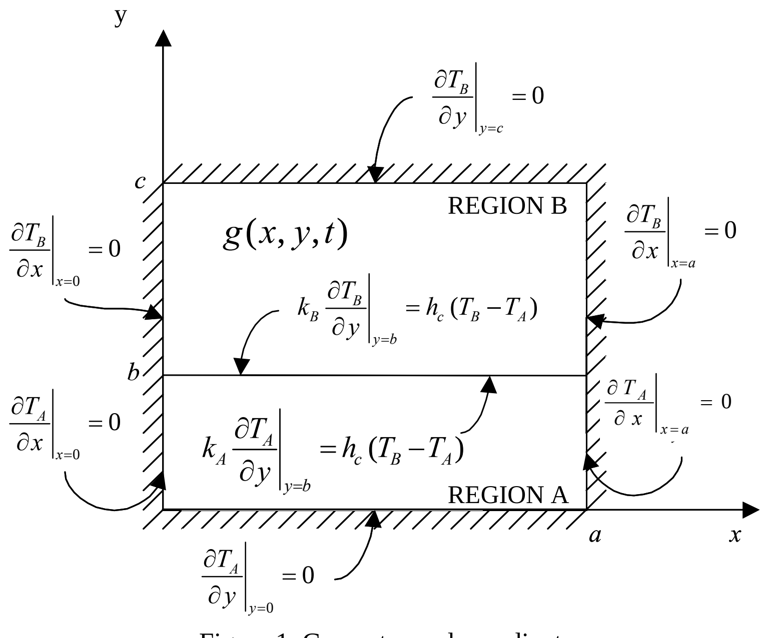

Figure 1. Geometry and coordinates.

In order to write eqns (1.a-f) and (2.a-f), the following dimensionless groups were defined:

$$\theta_A(X,Y,\tau)=\frac{T_A(x,y,t)-T_{0A}}{T_{0A}} \quad \theta_B(X,Y,\tau)=\frac{T_B(x,y,t)-T_{0A}}{T_{0A}} \quad \tau=\frac{\alpha_A\,t}{a^2} \quad X=\frac{x}{a} \quad Y=\frac{y}{a} \qquad (3.a\text{-}e)$$

$$G(X,Y,\tau)=\frac{g(x,y,t)a^2}{k_A T_{0A}} \quad Bi=\frac{h_c\,a}{k_A} \quad K=\frac{k_B}{k_A} \quad \Lambda=\frac{\alpha_B}{\alpha_A} \quad B=\frac{b}{a} \quad C=\frac{c}{a} \qquad (3.f\text{-}k)$$

where k is the thermal conductivity, α is the thermal diffusivity and the subscripts A and B refer to regions A and B, respectively.

3. DIRECT PROBLEM AND INVERSE PROBLEM

The problem defined by eqns (1.a-f) and (2.a-f), with known initial and boundary conditions, contact conductance, thermophysical properties and source term constitutes a *direct problem* that is concerned with the determination of the transient temperature fields $\theta_A(X,Y,\tau)$ and $\theta_B(X,Y,\tau)$.

For the *inverse problem* of interest here, the source term $G(X,Y,\tau)$ is regarded as unknown, while the other quantities appearing in the formulation of the direct problem are considered to be known with high degree of accuracy. For the solution of the inverse problem, we consider available the transient temperature measurements $\mu_{Am}(\tau)$ taken at the positions (X_m, Y_m), m=1,..,M, in region A, as well as $\mu_{Bn}(\tau)$, n=1,..,N, taken at the positions (X_n, Y_n) in region B. The measurements contain errors, which are supposed to be additive, uncorrelated, normally distributed, with known and constant standard deviation and zero mean.

For the estimation of the unknown function $G(X,Y,\tau)$, we make no *a priori* assumption regarding its functional form, except that it belongs to the Hilbert space of square-integrable functions [19-21] in the domain $0 < X < 1$, $B < Y < C$ and $0 < \tau < \tau_f$, where τ_f is the duration of the time interval of concern for the inverse analysis. For the solution of the present inverse problem, we consider the minimization of the following functional:

$$S[G(X,Y,\tau)] = \sum_{m=1}^{M} \int_{\tau=0}^{\tau_f} [\mu_{Am}(\tau) - \theta_A(X_m, Y_m, \tau; G)]^2 \, d\tau + \sum_{n=1}^{N} \int_{\tau=0}^{\tau_f} [\mu_{Bn}(\tau) - \theta_B(X_n, Y_n, \tau; G)]^2 \, d\tau \tag{4}$$

The minimization of the objective functional (4) is performed with the conjugate gradient method with adjoint problem formulation [19-21]. The iterative procedure of such method is presented below.

4. CONJUGATE GRADIENT METHOD

The iterative procedure of the conjugate gradient method, as applied to the estimation of the function $G(X,Y,\tau)$, is given by:

$$G^{k+1}(X,Y,\tau) = G^k(X,Y,\tau) - \beta^k \, d^k(X,Y,\tau) \tag{5.a}$$

where the superscript k denotes the number of iterations and β^k is the search step size. The direction of descent, d^k, is obtained as a linear combination of the gradient direction at iteration k with directions of descent at previous iterations. It is given as [19-21]:

$$d^k(X,Y,\tau) = \nabla S[G^k(X,Y,\tau)] + \gamma^k d^{k-1}(X,Y,\tau) \tag{5.b}$$

Different expressions are available in the literature for the conjugation coefficient, γ^k. In this work, we use the so-called Fletcher-Reeves version of the conjugate gradient method, where the conjugation coefficient is given in the form [19-21]:

$$\gamma^k = \frac{\int_{\tau=0}^{\tau_f} \int_{Y=B}^{C} \int_{X=0}^{1} \{\nabla S[G^k(X,Y,\tau)]\}^2 \, dX \, dY \, d\tau}{\int_{\tau=0}^{\tau_f} \int_{Y=B}^{C} \int_{X=0}^{1} \{\nabla S[G^{k-1}(X,Y,\tau)]\}^2 \, dX \, dY \, d\tau} \quad \text{for } k = 1,2,3,\ldots \quad \text{with } \gamma^0 = 0 \tag{5.c}$$

For the implementation of the iterative procedure of the conjugate gradient method given by eqns (5.a-c), expressions are required for the gradient direction $\nabla S[G^k(X,Y,\tau)]$, as well as for the search step size β^k. Two auxiliary problems are used to derive these expressions, namely the sensitivity and adjoint problems, as described next.

5. SENSITIVITY PROBLEM AND SEARCH STEP SIZE

The sensitivity problem is used to determine the variations $\Delta\theta_A(X,Y,\tau)$ and $\Delta\theta_B(X,Y,\tau)$ that the temperatures $\theta_A(X,Y,\tau)$ and $\theta_B(X,Y,\tau)$ undergo, respectively, when the unknown function is perturbed by $\Delta G(X,Y,\tau)$ [19-21]. The sensitivity problem is derived by substituting into the direct problem given by eqns (1.a-f) and (2.a-f),

$\theta_A(X,Y,\tau)$ by $[\theta_A(X,Y,\tau)+\Delta\theta_A(X,Y,\tau)]$, $\theta_B(X,Y,\tau)$ by $[\theta_B(X,Y,\tau)+\Delta\theta_B(X,Y,\tau)]$ and $G(X,Y,\tau)$ by $[G(X,Y,\tau)+\Delta G(X,Y,\tau)]$. The original direct problem is then subtracted from the resulting equations in order to obtain the following sensitivity problem:

Region A:

$$\frac{\partial \Delta\theta_A}{\partial \tau} = \frac{\partial^2 \Delta\theta_A}{\partial X^2} + \frac{\partial^2 \Delta\theta_A}{\partial Y^2} \quad \text{in } 0 < X < 1,\ 0 < Y < B, \text{ for } \tau > 0 \tag{6.a}$$

$$\left.\frac{\partial \Delta\theta_A}{\partial X}\right|_{X=0} = 0 \quad \text{at } X = 0,\ 0 < Y < B, \text{ for } \tau > 0 \tag{6.b}$$

$$\left.\frac{\partial \Delta\theta_A}{\partial X}\right|_{X=1} = 0 \quad \text{at } X = 1,\ 0 < Y < B, \text{ for } \tau > 0 \tag{6.c}$$

$$\left.\frac{\partial \Delta\theta_A}{\partial Y}\right|_{Y=0} = 0 \quad \text{at } Y = 0,\ 0 < X < 1, \text{ for } \tau > 0 \tag{6.d}$$

$$\left.\frac{\partial \Delta\theta_A}{\partial Y}\right|_{Y=B} = Bi(\Delta\theta_B - \Delta\theta_A) \quad \text{at } Y = B,\ 0 < X < 1, \text{ for } \tau > 0 \tag{6.e}$$

$$\Delta\theta_A(X,Y,0) = 0 \quad \text{for } \tau = 0, \text{ in } 0 < X < 1,\ 0 < Y < B \tag{6.f}$$

Region B:

$$\frac{1}{\Lambda}\frac{\partial \Delta\theta_B}{\partial \tau} = \frac{\partial^2 \Delta\theta_B}{\partial X^2} + \frac{\partial^2 \Delta\theta_B}{\partial Y^2} + \frac{\Delta G(X,Y,\tau)}{K} \quad \text{in } 0 < X < 1,\ B < Y < C, \text{ for } \tau > 0 \tag{7.a}$$

$$\left.\frac{\partial \Delta\theta_B}{\partial X}\right|_{X=0} = 0 \quad \text{at } X = 0,\ B < Y < C, \text{ for } \tau > 0 \tag{7.b}$$

$$\left.\frac{\partial \Delta\theta_B}{\partial X}\right|_{X=1} = 0 \quad \text{at } X = 1,\ B < Y < C, \text{ for } \tau > 0 \tag{7.c}$$

$$\left.\frac{\partial \Delta\theta_B}{\partial Y}\right|_{Y=B} = \frac{1}{K} Bi(\Delta\theta_B - \Delta\theta_A) \quad \text{at } Y = B,\ 0 < X < 1, \text{ for } \tau > 0 \tag{7.d}$$

$$\left.\frac{\partial \Delta\theta_B}{\partial Y}\right|_{Y=C} = 0 \quad \text{at } Y = C,\ 0 < X < 1, \text{ for } \tau > 0 \tag{7.e}$$

$$\Delta\theta_B(X,Y,0) = 0 \quad \text{for } \tau = 0, \text{ in } 0 < X < 1,\ B < Y < C \tag{7.f}$$

The search step size is obtained by minimizing the objective functional with respect to β^k at each iteration [19-21]. The following expression results:

$$\beta^k = \frac{\sum_{m=1}^{M}\int_{\tau=0}^{\tau_f}[\theta_A(X_m,Y_m,\tau;G)-\mu_{Am}(\tau)]\Delta\theta_A(X_m,Y_m,\tau;d^k)\,d\tau + \sum_{n=1}^{N}\int_{\tau=0}^{\tau_f}[\theta_B(X_n,Y_n,\tau;G)-\mu_{Bn}(\tau)]\Delta\theta_B(X_n,Y_n,\tau;d^k)\,d\tau}{\sum_{m=1}^{M}\int_{\tau=0}^{\tau_f}[\Delta\theta_A(X_m,Y_m,\tau;d^k)]^2\,d\tau + \sum_{n=1}^{N}\int_{\tau=0}^{\tau_f}[\Delta\theta_B(X_n,Y_n,\tau;d^k)]^2\,d\tau} \tag{8}$$

where $\Delta\theta_A(X,Y,\tau;d^k)$ and $\Delta\theta_B(X,Y,\tau;d^k)$ are the solutions of the sensitivity problem given by eqns (6.a-f) and (7.a-f), obtained by setting $\Delta G(X,Y,\tau)= d^k(X,Y,\tau)$.

6. ADJOINT PROBLEM AND GRADIENT EQUATION

The adjoint problem is derived by multiplying the governing equations of the direct problem, eqns (1.a) and (2.a,) by the Lagrange multipliers $\lambda_A(X,Y,\tau)$ and $\lambda_B(X,Y,\tau)$, respectively. The equations are then integrated in the spatial and time domains that they are valid and added to the original functional (4). The directional derivative of the extended functional in the direction of the perturbation of the unknown function is then obtained and the resultant expression, after some lengthy but straightforward manipulations, is allowed to go to zero [19-21]. The following adjoint problem for the determination of the Lagrange multipliers $\lambda_A(X,Y,\tau)$ and $\lambda_B(X,Y,\tau)$ results:

Region A:

$$-\frac{\partial \lambda_A}{\partial \tau} = \frac{\partial^2 \lambda_A}{\partial X^2} + \frac{\partial^2 \lambda_A}{\partial Y^2} + 2\sum_{m=1}^{M}[\theta_A(X,Y,\tau) - \mu_{Am}(\tau)]\delta(X - X_m)\delta(Y - Y_m)$$

in $0 < X < 1$, $0 < Y < B$, $0 < \tau < \tau_f$ (9.a)

$$\left.\frac{\partial \lambda_A}{\partial X}\right|_{X=0} = 0 \quad \text{at } X = 0,\ 0 < Y < B,\ 0 < \tau < \tau_f \qquad (9.b)$$

$$\left.\frac{\partial \lambda_A}{\partial X}\right|_{X=1} = 0 \quad \text{at } X = 1,\ 0 < Y < B,\ 0 < \tau < \tau_f \qquad (9.c)$$

$$\left.\frac{\partial \lambda_A}{\partial Y}\right|_{Y=0} = 0 \quad \text{at } Y = 0,\ 0 < X < 1,\ 0 < \tau < \tau_f \qquad (9.d)$$

$$\left.\frac{\partial \lambda_A}{\partial Y}\right|_{Y=B} = Bi\left(\frac{\lambda_B}{K} - \lambda_A\right) \quad \text{at } Y = B,\ 0 < X < 1,\ 0 < \tau < \tau_f \qquad (9.e)$$

$$\lambda_A(X,Y,\tau_f) = 0 \quad \text{for } \tau = \tau_f, \text{ in } 0 < X < 1,\ 0 < Y < B \qquad (9.f)$$

Region B:

$$-\frac{\partial \lambda_B}{\partial \tau} = \frac{\partial^2 \lambda_B}{\partial X^2} + \frac{\partial^2 \lambda_B}{\partial Y^2} + 2\sum_{n=1}^{N}[\theta_B(X,Y,\tau) - \mu_{Bn}(\tau)]\delta(X - X_n)\delta(Y - Y_n)$$

in $0 < X < 1$, $B < Y < C$, $0 < \tau < \tau_f$ (10.a)

$$\left.\frac{\partial \lambda_B}{\partial X}\right|_{X=0} = 0 \quad \text{at } X = 0,\ B < Y < C,\ 0 < \tau < \tau_f \qquad (10.b)$$

$$\left.\frac{\partial \lambda_B}{\partial X}\right|_{X=1} = 0 \quad \text{at } X = 1,\ B < Y < C,\ 0 < \tau < \tau_f \qquad (10.c)$$

$$\left.\frac{\partial \lambda_B}{\partial Y}\right|_{Y=B} = Bi\left(\frac{\lambda_B}{K} - \lambda_A\right) \quad \text{at } Y = B,\ 0 < X < 1,\ 0 < \tau < \tau_f \qquad (10.d)$$

$$\left.\frac{\partial \lambda_B}{\partial Y}\right|_{Y=C} = 0 \quad \text{at } Y = C,\ 0 < X < 1,\ 0 < \tau < \tau_f \qquad (10.e)$$

$$\lambda_B(X,Y,\tau_f) = 0 \quad \text{for } \tau = \tau_f, \text{ in } 0 < X < 1,\ B < Y < C \qquad (10.f)$$

By applying the limiting process used to obtain the adjoint problem, the directional derivative of the objective functional along the direction of the perturbation $\Delta G(X,Y,\tau)$ reduces to:

$$\Delta S[G(X,Y,\tau)] = \int_{\tau=0}^{\tau_f}\int_{Y=B}^{C}\int_{X=0}^{1} \frac{\lambda_B(X,Y,\tau)}{K}\Delta G(X,Y,\tau)\,dX\,dY\,d\tau \qquad (11.a)$$

We now invoke the hypothesis that the unknown function belongs to the Hilbert space of square integrable functions in the domain $0 < X < 1$, $B < Y < C$ and $0 < \tau < \tau_f$, so that we can write such directional derivative as:

$$\Delta S[G(X,Y,\tau)] = \int_{\tau=0}^{\tau_f}\int_{Y=B}^{C}\int_{X=0}^{1} \nabla S[G(X,Y,\tau)]\Delta G(X,Y,\tau)\,dX\,dY\,d\tau \qquad (11.b)$$

Therefore, by comparing eqns (11.a) and (11.b) we obtain the gradient direction as:

$$\nabla S[G(X,Y,\tau)] = \frac{\lambda_B(X,Y,\tau)}{K} \qquad (12)$$

After developing the expressions for the search step size and for the gradient direction, the iterative procedure of the conjugate gradient method given by eqns (5.a-c) can be applied until a suitable convergence criterion is satisfied. In this paper, the *discrepancy principle* [19-21] was used to specify the tolerance for the stopping criterion. In the discrepancy principle, the solution is assumed to be sufficiently accurate when the difference between measured and estimated quantities is of the order of magnitude of the measurement errors.

7. RESULTS AND DISCUSSIONS

We now examine the solution of the inverse problem of estimating the source term function $G(X,Y,\tau)$ by using simulated temperature measurements containing random errors. For the results presented below, the dimensions of regions A and B were taken as a = 0.3 m, b = 0.015 m and $(b\text{-}c)$ = 0.1 m, so that B = 0.05 and C = 0.38. In order to examine strict cases involving materials with thermal conductivity and thermal diffusivity of different orders of magnitudes, we assumed regions A and B to be made of Teflon and steel, respectively, with the following thermophysical properties: k_A = 0.23 W/mK, α_A = 1.005 x 10^{-7} m^2/s, k_B = 45 W/mK and α_B= 1.240 x 10^{-5} m^2/s, that is, K = 195.65 and Λ = 123.35. Both regions A and B were supposed to be initially at the same temperature T_{0A} = T_{0B} = 27 ^{o}C. Perfect contact between the two regions was assumed for the results presented hereafter. The duration of the experiment was taken as 2340 s, with a measurement frequency of 1.1 Hz. In dimensionless terms, the duration of the experiment was τ_f = 2.613 x 10^{-3}. However, in order to overcome the difficulties encountered in the inverse problem solution in the neighborhood of the final time, which result from the null gradient at $\tau = \tau_f$, see eqns (9.f), (10.f) and (12), the results are presented below up to τ =2x10^{-3}. The sensors were supposed to be located only within region A, which constitutes a difficult situation for the estimation of the source term in region B. For the cases examined below, the sensors were evenly distributed at the vertical position Y = 0.03, which corresponds to 0.006 m below the interface between regions A and B. The simulated measurements contained random errors with standard deviation of 0.003, which is equivalent to 1^{o}C. Such simulated measurements were generated by using functions containing discontinuities and sharp-corners for the source term. These functions were supposed to vary from zero to a maximum value of 10^6 W/m^3. The initial guess for the iterative procedure of the conjugate gradient method was taken as a null source term.

The direct, sensitivity and adjoint problems were solved numerically by using the finite-volume method. Regions A and B were discretized with 30 volumes along the X direction and 25 and 50 volumes along their thickness, respectively. A step of $\Delta\tau$ =10^{-6} was used to march the solution in time. Such numbers of volumes and time step were selected though a grid convergence analysis and by comparing the numerical and analytical solutions for the direct problem.

Figures 2.a-d present the results for the time variation of a spatially uniform source term, obtained by using 5 sensors. Figures 2.a-d present the results obtained for the positions Y = 0.05, 0.16, 0.27 and 0.38, respectively, and for each of these figures the results for X = 0.02, 0.22, 0.42 and 0.98 are given. We notice in Figures 2.a-d that the accuracies of the estimated functions deteriorate for vertical positions closer to the interface at Y=0.05, or closer to the upper boundary at Y=0.38. On the other hand, the estimated functions are very little affected by the longitudinal position X, except for Y=0.05. At Y=0.05, the estimated functions are less accurate in regions near the surfaces at X=0 and X=1. It is interesting to notice in Figures 2.a-d that the differences between estimated and exact functions, in the regions where the exact function is underestimated, are practically identical to those differences in the regions where the exact function is overestimated. Therefore, the total energy input is accurately estimated, despite the fact that the source term functional form is not exactly recovered for some cases.

Figures 3.a-d present the results obtained for the same exact function of Figures 2.a-d, but for the measurements of 10 sensors instead of 5. A comparison of Figures 2 and 3 shows that, as expected, the solution improves as the measurements of more sensors are used, since more information is available for the inverse analysis.

We now examine a test case involving the identification of a source term that varies along the X direction, but is constant in time and uniform along the Y direction. Figures 4.a-c present the results obtained for the estimation of a source term containing sharp corners for τ = 0.0001, τ = 0.001 and τ = 0.002, respectively, obtained by using 10 sensors in the inverse analysis. In each of these figures the results for Y = 0.05, 0.16, 0.27 and 0.38 are given. These figures show that the accuracy of the inverse problem solution deteriorates for the positions located farther from the interface, because the measurements are less affected by the source term in such locations. However, as for the results shown in Figures 2 and 3, the total energy input was accurately estimated. The inverse problem solution was not affected by the time variable in this case, even for τ = 0.002, which is near the final time considered for the problem.

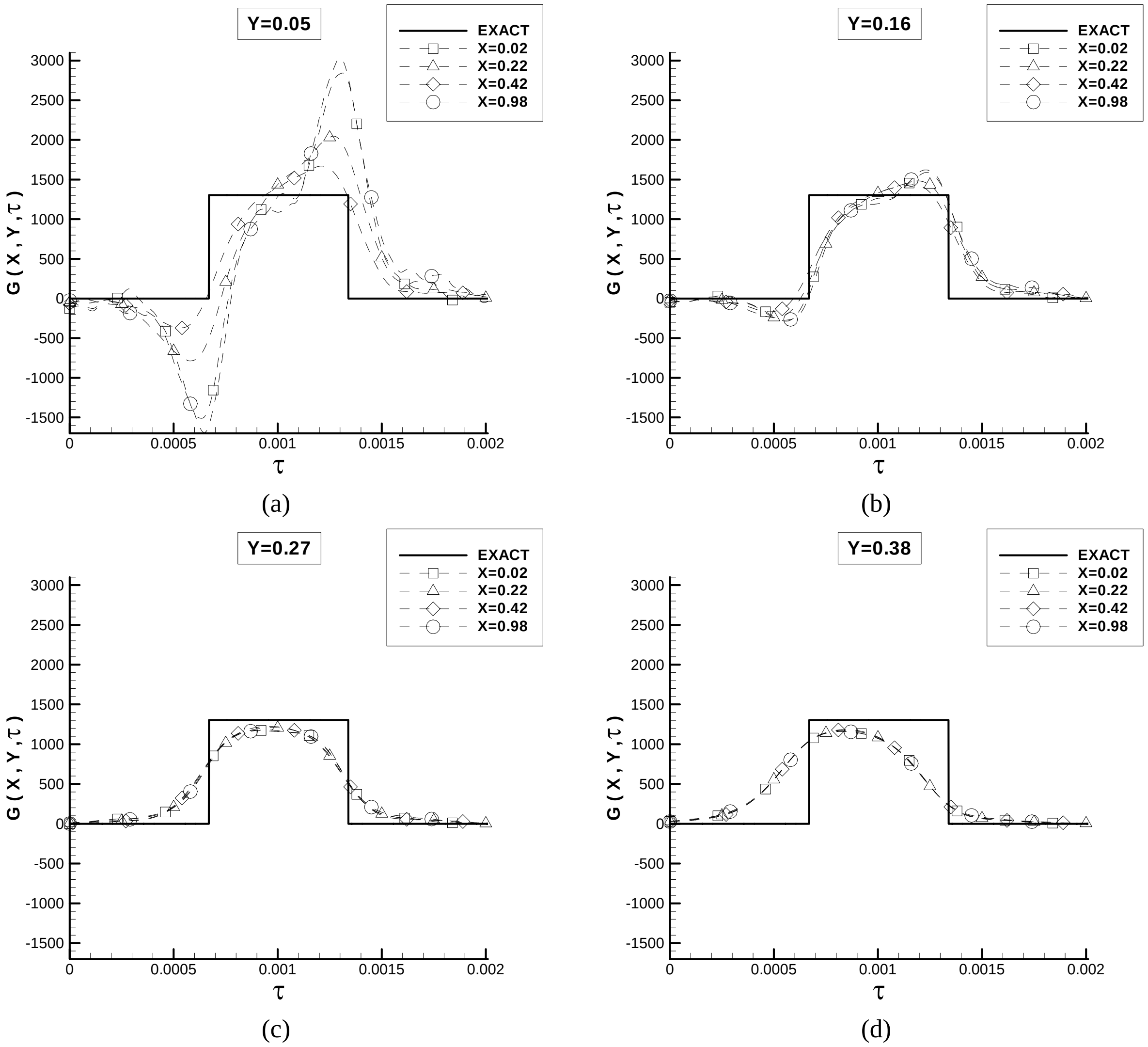

Figure 2. Inverse problem solution obtained with 5 sensors for a uniform exact function containing discontinuities in time: (a) $Y = 0.05$, (b) $Y = 0.16$, (c) $Y = 0.27$ and (d) $Y = 0.38$.

The results for a source term that varies in time and along the longitudinal X direction, but is uniform along the Y direction, are presented in Figures 5.a-c for $\tau = 0.0001$, $\tau = 0.001$ and $\tau = 0.002$, respectively. Such results were obtained with the measurements of 10 sensors. As for the case involving only the variation of the source term along the X direction (see Figures 4.a-c), we notice in Figures 5.a-c that the results for vertical positions close to the interface are more accurate. Furthermore, the results for this case become less accurate as the final time is approached (see figure 5.c, for $\tau = 0.002$), when the estimated function is affected by the null gradient.

We also examined in this work cases involving the identification of source terms varying along the vertical Y direction. However, the exact function could not be recovered unless measurements were considered available within region B as well. Such was the case because temperature measurements taken within region A were not sensitive to variations of the source term along the Y direction in region B. In fact, the results presented above reveals that the agreement between estimated and exact functions deteriorates for the positions farther from the interface, even when the source term was uniform along the Y direction.

8. CONCLUSIONS

This work dealt with the solution of the inverse problem of estimating the source term function in a rectangular region, by using simulated temperature measurements taken within another contacting region. The conjugate gradient method of function estimation was applied for the solution of the inverse problem. The materials considered for the two regions involved quite different thermal properties. Several test cases were examined in the paper, including functions varying only in time, as well as varying spatially and in time.

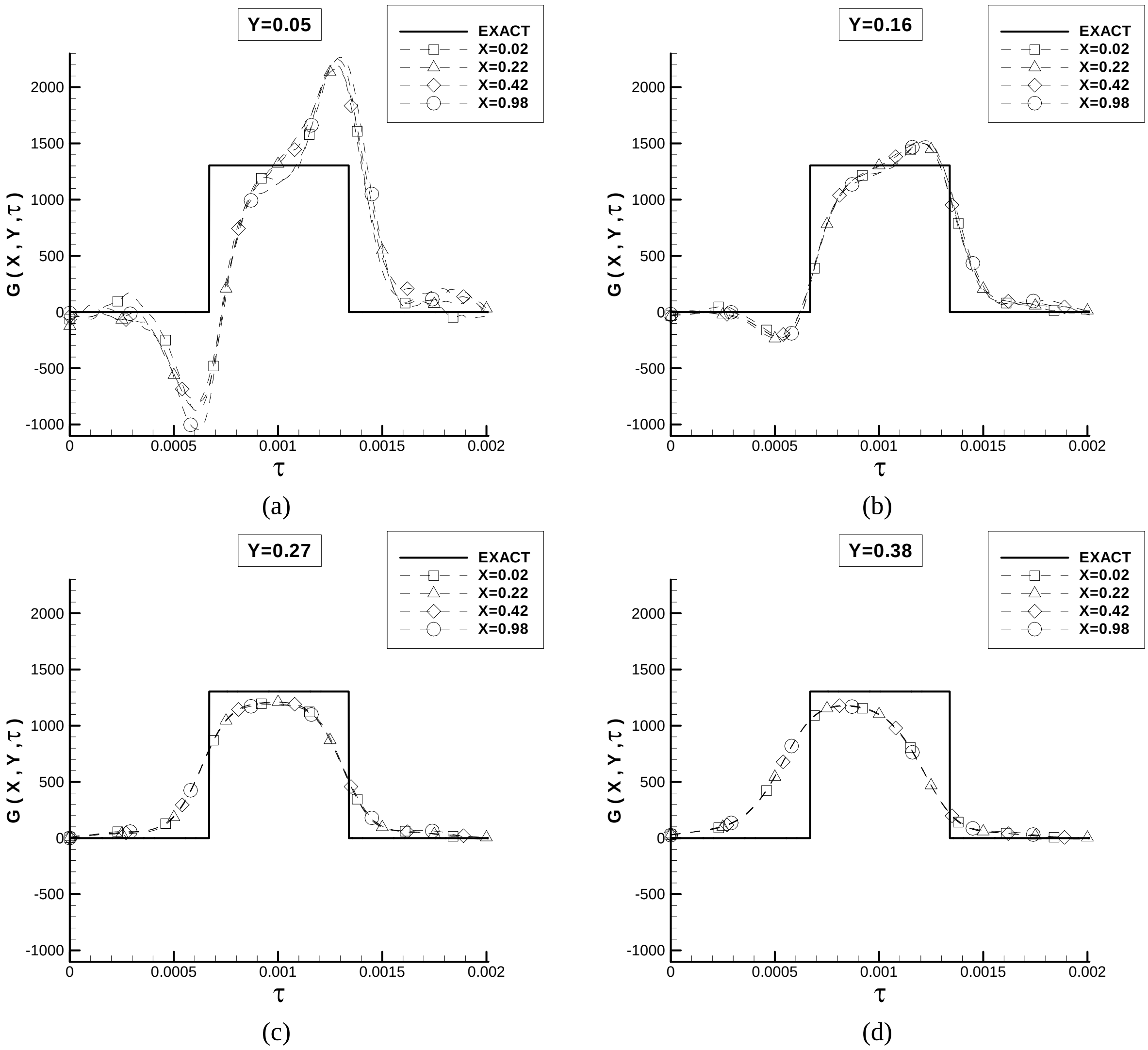

Figure 3. Inverse problem solution obtained with 10 sensors for a uniform exact function containing discontinuities in time: (a) $Y = 0.05$, (b) $Y = 0.16$, (c) $Y = 0.27$ and (d) $Y = 0.38$.

The results obtained here show that the present function estimation approach was capable of recovering spatially and time dependent functions. Generally, the agreement between estimated and exact functions improves for those locations closer to the interface between the two regions.

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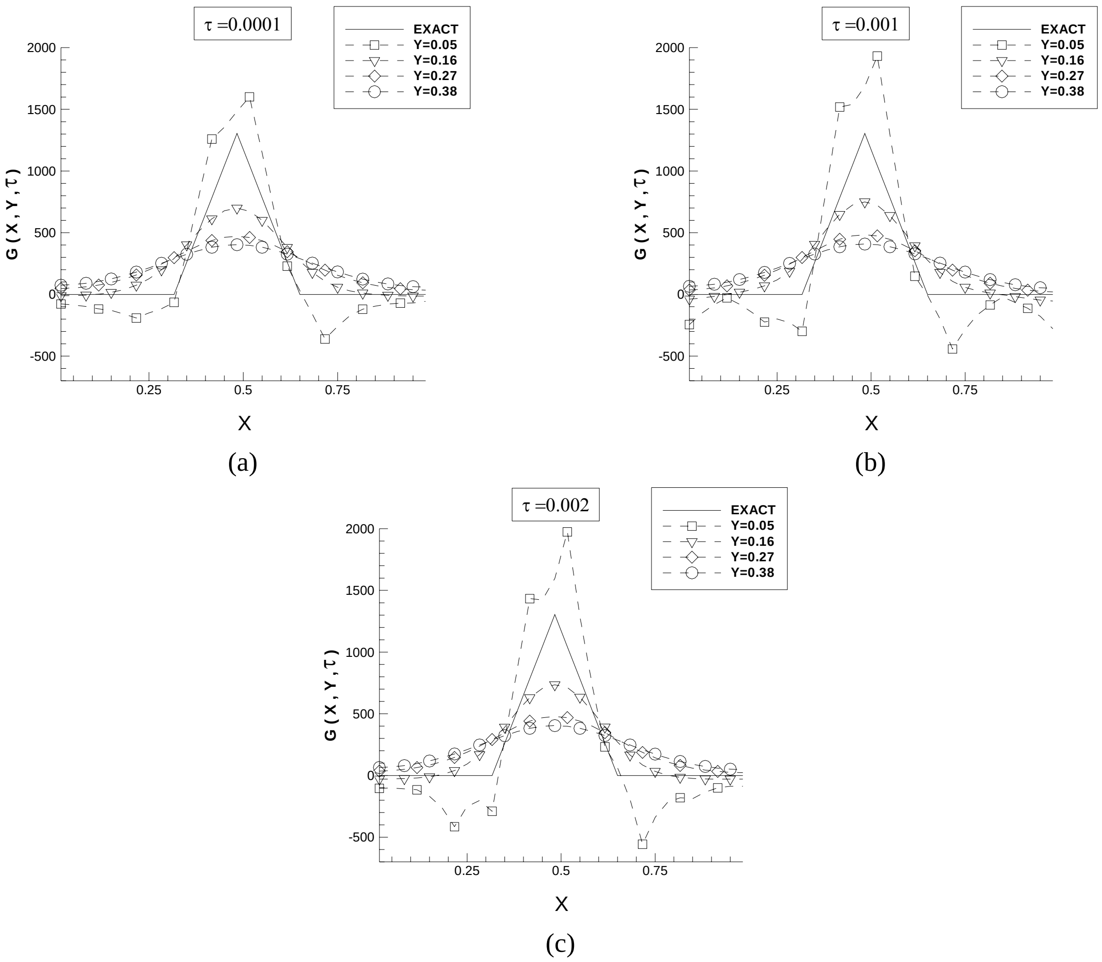

Figure 4. Inverse problem solution obtained with 10 sensors for an exact function varying along the longitudinal direction: (a) $\tau = 0.0001$, (b) $\tau = 0.001$ and (c) $\tau = 0.002$.

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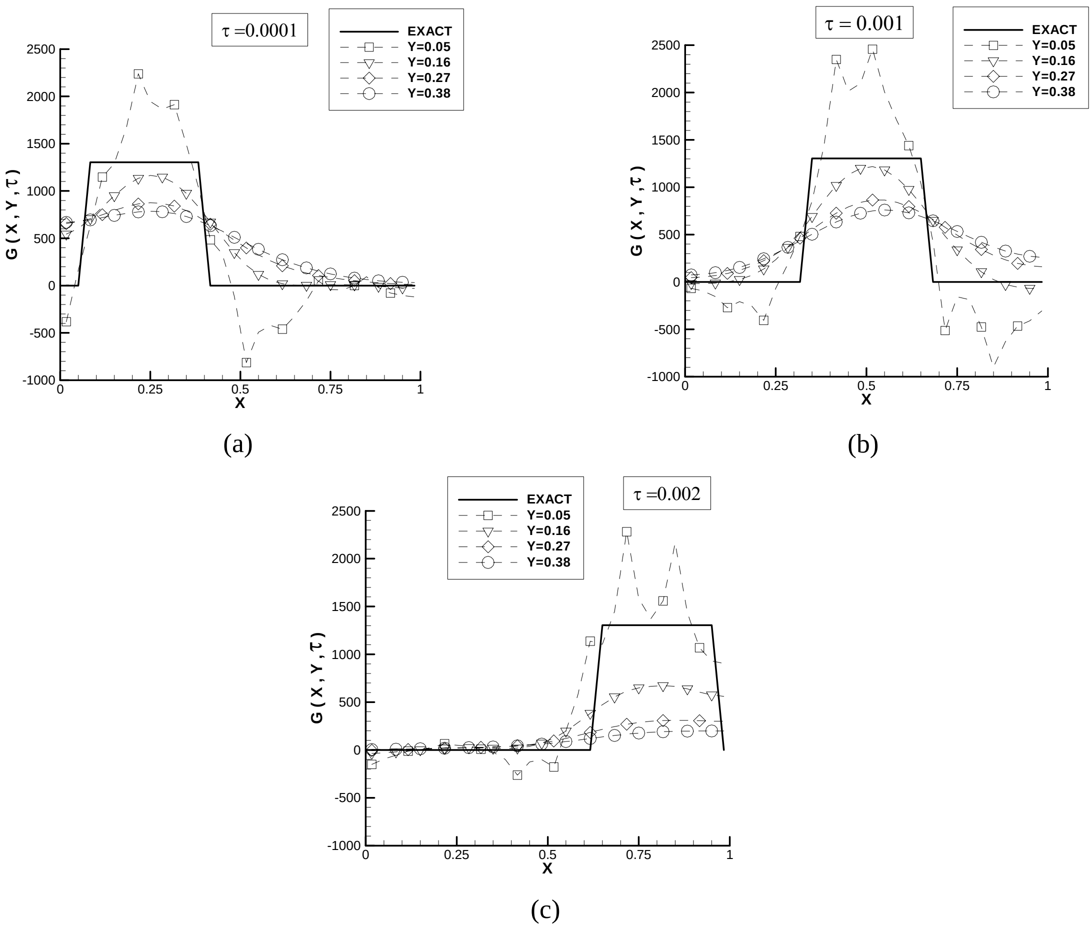

Figure 5. Inverse problem solution obtained with 10 sensors for an exact function varying in time and along the longitudinal direction: (a) $\tau = 0.0001$, (b) $\tau = 0.001$ and (c) $\tau = 0.002$.

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